Complex Modelling In The Primary/Middle School Years

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Abstract The world's increasing complexity, competitiveness, interconnectivity, and dependence on technology generate new challenges for nations and individuals that cannot be met by continuing education as usual (Katehi, Pearson, & Feder, 2009). With the proliferation of complex systems have come new technologies for communication, collaboration, and conceptualisation. These technologies have led to significant changes in the forms of mathematical and scientific thinking that are required beyond the classroom. Modelling, in its various forms, can develop and broaden children's mathematical and scientific thinking beyond the standard curriculum. This paper first considers future competencies in the mathematical sciences within an increasingly complex world. Next, consideration is given to interdisciplinary problem solving and models and modelling. Examples of complex, interdisciplinary modelling activities across grades are presented, with data modelling in 1st grade, model-eliciting in 4th grade, and engineering-based modelling in 7th-9th grades.

1. Introduction

In recent decades our global community has rapidly become a knowledge driven society, one that is increasingly dependent on the distribution and exchange of services and commodities, and one that has become highly inventive where creativity, imagination, and innovation are key players. At the same time, the world has become governed by complex systems—financial corporations, the World Wide Web, education and health systems, traffic jams, and classrooms are just some of the complex systems we deal with on a regular basis. For all citizens, an appreciation and understanding of the world as interlocked complex systems is critical for making effective decisions about one's life as both an individual and as a community member (Bar-Yam, 2004; Jacobson & Wilensky, 2006; Lesh, 2006).

Complexity—the study of systems of interconnected components whose behavior cannot be explained solely by the properties of their parts but from the behaviour that arises from their interconnectedness—is a field that has led to significant scientific methodological advances. With the proliferation of complex systems has come new technologies for communication, collaboration, and conceptualisation. These technologies have led to significant changes in the forms of mathematical and scientific thinking that are needed beyond the classroom, such as the need to generate, analyse, operate on, and transform complex data sets (English & Sriraman, 2010).

Educational leaders from different walks of life are emphasising the importance of developing students' abilities to deal with complex systems for success beyond school. Such abilities include: constructing, describing, explaining, manipulating, and predicting complex systems; working on multi-phase and multi-component projects in which planning, monitoring, and communicating are critical for success; and adapting rapidly to ever-evolving conceptual tools (or complex artifacts) and resources (Gainsburg, 2006; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

2. Future Competencies in the Mathematical Sciences

The Australian Government's national key reform initiatives in education demonstrate a commitment to building a stronger foundation of mathematics and science in schools and universities (DEEWR, 2010). Numerous concerns have been expressed over students' achievements in the sciences (e.g., Australian Academy of Science, 2006, 2010; Business Council of Australia, 2007), with the latter emphasising that "too many young Australians are being left behind by our school education system," beginning with early learning in mathematics and science. The Business Council of Australia argues that many aspects of our school system "have not changed since the 1960s." Likewise, other nations are highlighting the need for a renaissance in the science, technology, engineering, and mathematics (STEM) fields (e.g., The National Academies, USA, 2009). Indeed, the first recommendation of The National Academies' Rising above the Gathering Storm (2007, 2009) was to vastly improve K-12 science and mathematics education.

With the advent of digital technologies have come changes in the future world of work for our students. As Clayton (1999) and others (e.g., Jenkins, Clinton, Purushotma, Robinson, & Weigel, 2006; Lombardi & Lombardi, 2007) have stressed, the availability of increasingly sophisticated technology has led to changes in the way mathematics and science are being applied in work place settings. These technological changes have led to both the addition of new competencies and the elimination of existing skills, together with increased application of interdisciplinary knowledge in solving problems and communicating results. Although we cannot simply list a number of competencies and assume these can be automatically applied to the workplace setting, there are several that employers generally consider to be essential to productive outcomes (e.g., Doerr & English, 2003; English, 2008; Gainsburg, 2006; Lesh & Zawojewski, 2007). In particular, the following are some of the core competencies that have been identified as key elements of productive and innovative work place practices (English, Jones, Bartolini Bussi, Lesh, Tirosh, & Sriraman, 2008).

- Problem solving, including working collaboratively on complex problems where planning, overseeing, moderating, and communicating are essential elements for success;
- Applying numerical and algebraic reasoning in an efficient, flexible, and creative manner;
- Generating, analysing, operating on, and transforming complex data sets;
- Applying an understanding of core ideas from ratio and proportion, probability, rate, change, accumulation, continuity, and limit;
- Constructing, describing, explaining, manipulating, and predicting complex systems;
- Thinking critically and being able to make sound judgments, including being able to distinguish reliable from unreliable information sources;
- Synthesizing, where an extended argument is followed across multiple modalities;
- Engaging in research activity involving the investigation, discovery, and dissemination of pertinent information in a credible manner;
- Flexibility in working across disciplines to generate innovative and effective solutions;
- Techno-mathematical literacy ("where the mathematics is expressed through technological artefacts" Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2010, p. 14).

3. Interdisciplinary Problem Solving

These future competencies alert us to rethink the nature of the learning tasks we implement in our classrooms. I argue that we need a focus on future-oriented, interdisciplinary problem-solving experiences, which mirror problem solving beyond the classroom. This focus is especially needed, given that "problems themselves change as rapidly as the professions and social structures in which they are embedded change" (Hamilton, 2007, p. 2). For example, experiences that draw upon the broad field of engineering provide powerful links between the classroom and the real world, enabling students to apply their mathematics and science learning to the solution of authentic problems (English & Mousoulides, 2011; Kuehner & Mauch, 2006).

Our challenge then, is how to promote creative and flexible use of mathematical and scientific ideas within an interdisciplinary context where students solve substantive, authentic problems that address multiple core learnings. One approach is through modelling involving cycles of model construction, evaluation, and revision, which is fundamental to mathematical and scientific understanding and to the professional practice of mathematicians and scientists (Lesh & Zawojewski, 2007; Romberg, Carpenter, & Kwako, 2005). Modelling is not just confined to mathematics and science, however. Other disciplines including engineering, economics, information systems, social and environmental science, and the arts have also contributed in large part to the powerful mathematical models we have in place for dealing with a range of complex problems (Beckmann, Michelsen, & Sriraman, 2005). Unfortunately, many mathematics and science curricula do not capitalise on the contributions of other disciplines. A more interdisciplinary and unifying model-based approach to students' mathematics learning could go some way towards alleviating the well-known "one inch deep and one mile wide" problem in many of our curricula (Sabelli, 2006, p. 7; Sriraman & Steinthorsdottir, 2007). There is limited research, however, on ways in which we might incorporate other disciplines within the mathematics curriculum.

3.1 Models and Modelling

Modelling is increasingly recognised as a powerful vehicle for not only promoting students' understanding of a wide range of key mathematical and scientific concepts, but also for helping them appreciate the potential of the mathematical sciences as a critical tool for analysing important issues in their lives, communities, and society in general (Greer, Verschaffel, & Mukhopadhyay, 2007; Romberg et al., 2005). Importantly, modelling needs to be integrated within the primary school curriculum and not reserved for the secondary school years and beyond as it has been traditionally. Research has shown that primary school children are indeed capable of engaging in modelling (e.g., English & Watters, 2005).

The terms, models and modelling, have been used variously in the literature, including in reference to solving word problems, conducting mathematical simulations, creating representations of problem situations (including constructing explanations of natural phenomena), and creating internal, psychological representations while solving a particular problem (e.g., English & Halford, 1995; Gravemeijer, 1999; Lesh & Doerr, 2003; Romberg et al., 2005). One perspective on models that I have adopted in my research is that of conceptual systems or tools comprising operations, rules and relationships that can describe, explain, construct, or modify an experience or a complex series of experiences. Modelling involves the crossing of disciplinary boundaries, with an emphasis on the structure of ideas, connected forms of knowledge, and the adaptation of complex ideas to new contexts (Hamilton, Lesh, Lester, & Brilleslyper, 2008). The modelling problems I implement in classrooms are realistically complex situations where the problem solver engages in mathematical and scientific thinking beyond the usual school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal (Lesh & Zawojewski, 2007).

In the remainder of this paper, I describe some interdisciplinary modelling activities that I have implemented in first grade (data modelling, focusing on caring for the environment), fourth grade (model-eliciting activity addressing the study of society and the environment), and seventh-ninth grades (engineering-based modelling experiences).

4. Data Modelling in First Grade

I have been conducting a longitudinal study of data modelling in grades 1-3 (2009-2011), where children engage in multiple, real-life experiences that incorporate other disciplines, such as health and nutrition, and environmental studies. The children investigate meaningful phenomena, decide what is worthy of attention (identifying complex attributes), and then progress to organising, structuring, visualising, and representing data (English, 2010a). Identifying variation, drawing inferences, and making predictions are also important components of data modelling in the early years (Watson, 2006).

In designing the classroom activities, literature was used as a basis for the problem context. It is well documented that storytelling provides an effective context for mathematical learning, with children being more motivated to engage in mathematical activities and displaying gains in achievement (van den Heuvel-Panhuizen & Van den Boogaard, 2008). Picture story books that addressed the overall theme of *Looking after our Environment*, a key theme in the teachers' curriculum at the time, were selected.

To illustrate the data modelling activities being implemented, the second and third of four activities of the first year of the study are described, namely, *Fun with Michael Recycle* and *Litterbug Doug*. The Australian picture story books that served as the basis for these activities were *Michael Recycle* (Bethel, 2008) and *Litterbug Doug* (Bethel, 2009). The former tells the story of Michael Recycle who came from the sky to clean up a very dirty town, with his motto, "I'm green and I'm keen to save the planet." Litterbug Doug was originally a very dirty creature who lived in a pile of rubbish in a very clean town. A "green-caped crusader" then swooped to the Earth to reform Litterbug Doug. As a consequence, Litterbug Doug became the Litter Police for the town and enthusiastically monitored the town's environment.

Fun with Michael Recycle, involved two lessons (lesson one, average duration of 30 minutes and lesson two, 60 minutes). The activity involved posing questions, identifying and generating attributes, organising and analysing data, and displaying and representing data in different ways.

Prior to the lessons, the storybook, *Michael Recycle*, was read and discussed, and one teacher's classroom (which was used in turn by the three classes) was set up with collections of reusable/recyclable and waste items. Next, each child in each group was given two Post-It notes and the group was directed to explore the classroom for these various items. Each group member was to draw and name an item on each Post-It note. The groups subsequently returned to their group desk and proceeded to discuss the attributes of their items, then organise, analyse, and represent their data however they chose (on a large sheet of paper provided.). On completion, the groups reported back to the class on how they represented their data. A brief whole class discussion followed on the nature of the attributes the children had identified and how they had organised and represented their data.

Following this, the children were advised that Michael Recycle "really likes the different ways you have represented your recyclable/reusable and waste items but would like you to represent them in a different way on your chart paper." The children were given a second sheet of paper to do so and were to leave their initial representation sheet intact. On completion, the groups reported back to the class, during which they were encouraged to explain their new representation and indicate how it differed from their first.

The second activity, *Litterbug Doug*, was designed to engage the children in interpreting tables of data, identifying variations in the data, posing questions, and making predictions. The activity was implemented in one lesson, average duration of 75 minutes. Prior to the lesson, the children read and discussed the storybook, *Litterbug Doug*. The lesson began with the teacher explaining that "Now that Litterbug Doug has become the Litter Police, the townsfolk are interested to see what he collects in Central Park during his first three days. They also want to know if Litterbug Doug is doing a good job of collecting litter in Central Park." The children were then shown a table displaying how many of each of five items Litterbug Doug collected on day 1, with the explanation that "As a start, the town's mayor asked Litterbug Doug to show him what he collected on his first day, Monday. Litterbug Doug showed the mayor what he saw and what he collected in the park." Next, the children posed questions to explore their interpretation of the table, given that they had had almost no exposure to such a table. It was then explained to the children that "Litterbug Doug has now collected litter in Central Park for three days and the townsfolk are keen to see how much he has collected." The children were then presented with a second table showing how many items had been collected from Monday through to Wednesday, with the Thursday column left blank. In their groups, children were to explore the second table, first noting the numbers of items collected on the second and third days, then how the data varied across the first three days and why this might be the case. Their next task was to consider the blank Thursday column. The children were to predict how many different items Litterbug Doug might have collected on Thursday. On completion, the groups reported back to the class on the variation they noticed in the data and on their predictions for Thursday.

5. Model-Eliciting Activity in the Primary School

As previously noted, mathematical modelling provides an ideal vehicle for interdisciplinary learning as the problems draw on contexts and data from other domains. One problem (a model-eliciting activity) that I implemented in fourth-grade classes, namely, *The First Fleet*, complemented the children's study of Australia's settlement and was implemented in four 50-minute sessions (English, 2010b).

First, the children were presented with background information on the problem context, namely, the British government's commissioning of 11 ships in May, 1787 to sail to "the land beyond the seas." The children answered a number of "readiness questions" to ensure they had understood this background information. After responding to these questions, the children were presented with the problem itself, together with a table of data listing 13 key environmental elements to be considered in determining the suitability of each of five given sites. The children were also provided with a comprehensive list of the tools and equipment, plants and seeds, and livestock that were on board the First Fleet. The problem text explained that, on his return from Australia to the UK in 1770, Captain James Cook reported that Botany Bay had lush pastures and well watered and fertile ground suitable for crops and for the grazing of cattle. But when Captain Phillip arrived in Botany Bay in January 1788 he thought it was unsuitable for the new settlement. Captain Phillip headed north in search of a better place for settlement. The children's task was as follows:

Where to locate the first settlement was a difficult decision to make for Captain Phillip as there were so many factors to consider. If you could turn a time machine back to 1788, how would you advise Captain Phillip? Was Botany Bay a poor choice or not? Early settlements occurred in Sydney Cove Port Jackson, at Rose Hill along the Parramatta River, on Norfolk Island, Port Hacking, and in Botany Bay. Which of these five sites would have been Captain Phillip's best choice? Your job is to create a system or model that could be used to help decide where it was best to anchor their boats and settle. Use the data given in the table and the list of provisions on board to determine which location was best for settlement. Whilst Captain Phillip was the first commander to settle in Australia many more ships were planning to make the journey and settle on the shores of Australia. Your system or model should be able to assist future settlers make informed decisions about where to locate their townships.

The children worked the problem in small groups, with no direct teaching from the teachers or researchers. In the final session, the children presented group reports on their models to their peers, who, in turn, asked questions about the models and gave constructive feedback.

6. Engineering-Based Modelling Experiences

In collaboration with an engineering educator and a science educator, I have been implementing engineering-based experiences in grades 7-9 (e.g., English, Hudson, & Dawes, 2010). In the first year of the study, we commenced by introducing the students to the varied world of engineering, including the different roles of engineers (two lessons of approx 45 minutes duration). This was followed by 5-7 lessons that explored bridges and their construction. These lessons entailed: learning about the work of civil engineers; exploring bridge structure with a focus on the main types of bridges in Brisbane, the students' home city; recognising features/constraints of the main bridge types; and investigating the concepts of tension, compression, load distribution, reinforcement, and strength, and their importance in bridge designs. Next, the students were to plan, design, model, and construct a truss bridge with given constraints and materials. The activity was set within the context of the students assisting two engineer graduates, as follows:

Remember our engineer graduates, Ben and Jane and our need for a bus bridge across the Brisbane River from Adelaide St to Southbank? Well you are now going to help them design and build a model bridge that will solve the transport problem near Victoria Bridge. Engineers always consider their design objective when creating their models. Also, they often have many design constraints or limitations they have to take into consideration. Ben and Jane's design objective is to make a truss bridge that:

1. Can span a distance of 150 metres.
2. Must support the most weight for the vehicles that will pass over it.
3. Must not disturb the river's fish.
4. Must not obstruct normal watercraft, such as the City Cat.
5. Must be at least 12 metres above water level.

The students were given a limited number of resources to build their bridge (drinking straws, sticky tape, scissors, rulers, small containers, metal washers). On completion of their bridge, the students reported back to the class explaining their steps to designing, modelling, and building their bridge. They were to explain how they used engineering design processes, namely, define the problem, brainstorm, select the most promising design, communicate the design, create and test the design, and evaluate and revise the design. Finally, the students were to indicate how they might have improved their design to strengthen their bridge.

7. Discussion and Conclusions

The modelling activities described here provide students with opportunities to repeatedly express, test, and refine or revise their current ways of thinking. The problems are designed so that multiple solutions of varying mathematical and scientific sophistication are possible and students with a range of personal experiences and knowledge can participate. The products students create are documented, shareable, reusable, and modifiable models that provide teachers with a window into their students' conceptual understanding. Furthermore, these modelling problems build communication (oral and written) and teamwork skills, both of which are essential to success beyond the classroom.

One of our main challenges in promoting complex learning through future-oriented problem solving is to find ways to utilise the powerful competencies developed in the early school years as a springboard for further mathematical and scientific power as students progress through the grade levels. I offer three interrelated suggestions for addressing this challenge: (a) Recognise that learning is based within contexts and environments that we, as educators shape, rather than within children's maturation (Lehrer & Schauble, 2007); (b) Design activities that promote learning across disciplines; and (c) Create learning activities that are of a high cognitive demand (Silver, Mesa, Morris, Star, & Benken, 2009). While not elaborating further on these suggestions, I believe any such activities that encourage complex learning should engage students in knowledge generation and active processing (Curious Minds, 2008). Recent research has argued for students to be exposed to learning situations in which they are not given all of the required mathematical and scientific tools, but rather, are required to create their own versions of the tools as they determine what is needed (e.g., English & Sriraman, 2010; Hamilton, 2007; Lesh, Hamilton, & Kaput, 2007). In particular, there are four features that I consider especially important in advancing students' mathematical and scientific learning, as indicated in Figure 1.

The need to incorporate future-oriented understandings and competencies within students' experiences in the mathematical sciences has never been greater. Intellectually stimulating activities that draw upon multidisciplinary content and contexts are paramount. Indeed, it is worth highlighting the words of Greer and Mukhopadhyay (2003): they commented that "the most salient features of most documents that lay out a K-12 program for mathematics education is that they make an intellectually exciting program boring," a feature they refer to as "intellectual child abuse" (p. 4). Clearly, we need to make the mathematical and scientific experiences we create for our students more

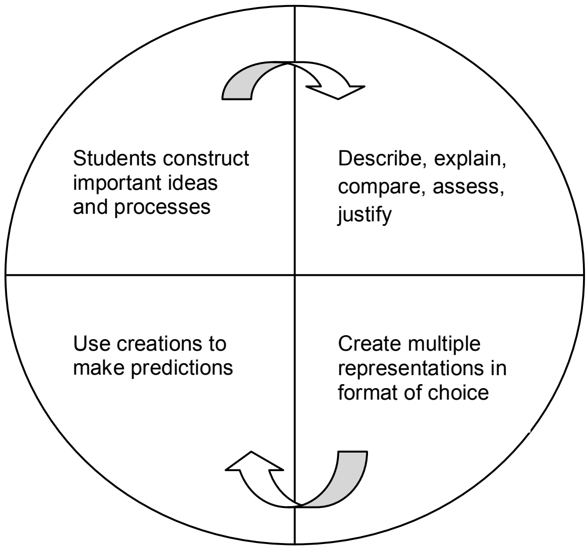

Fig. 1 Advancing Complex Learning

challenging, authentic, and meaningful. Developing students' abilities to work creatively with and generate mathematical and scientific knowledge, as distinct from working creatively on tasks that provide the required knowledge (Bereiter & Scardamalia, 2006), is especially important in preparing our students for success in a knowledge-based economy.

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